TEMPERATURE FIELD PRODUCED BY A SURFACE SOURCE ON AN INFINITE CYLINDER

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The temperature field produced in an infinite circular cylinder on application of a temperature step to a bounded portion of its surface is analyzed by successive application of Fourier and Laplace transformations. Curves are given showing the time dependence of the axial and radial temperature distributions for a temperature step applied to a narrow annular portion of the surface.

In the present article we analyze the temperature field produced in an infinite cylinder ($0 \le \rho \le a$, $0 \le \varphi \le 2\pi$, $-\infty < z < \infty$) when a bounded portion of its surface is instantaneously raised to a temperature T_0 which is subequently maintained constant. The temperature of the rest of the surface of the cylinder is taken to be always zero.

We thus require to solve the boundary problem

$$\Delta T = \frac{1}{c}\frac{\partial T}{\partial t}, \tag{1}$$

where $\Delta = (\partial^2/\partial\rho^2) + (1/\rho)(\partial/\partial\rho) + (1/\rho^2)(\partial^2/\partial\varphi^2) + (\partial^2/\partial z^2)$ is the Laplace operator, and c is the thermal conductivity. For $t \le 0$ the temperature $T = 0$ within and on the surface of the cylinder, and for $t > 0$

$$T_{\rho=a} = \begin{cases} \dfrac{N}{4bh}, & \text{when } \begin{cases} |z| \leqslant h \\ |\varphi| \leqslant \alpha \end{cases}, \\ 0 \text{ for all other z and } \varphi, \end{cases} \tag{2}$$

where $N/4bh = T_0 = \text{const}$; N is a constant of suitable dimensionality; 2h is the linear extent of the source as measured along the cylinder (at $\rho = a$); and $2b = 2\alpha a$ is the linear extent of the source as measured around the perimeter of the cylinder.

Further, the function T and its derivatives are required to approach zero as $|z| \to \infty$.

We express condition (2) in the form of a Fourier series:

$$T_{\rho=a} = \begin{cases} \dfrac{N}{4bh}\displaystyle\sum_{n=0}^{\infty}\frac{\sin(n\alpha)\cos(n\varphi)}{\pi\varepsilon_n n} & \text{for } |z| \leqslant h, \\ 0 & \text{for } |z| > h, \end{cases}$$

where $\varepsilon = 2$ for $n = 0$ and $\varepsilon = 1$ for $n = 1, 2, 3, \ldots$

We now apply in succession in Eq. (1) and condition (2) a Fourier transformation [1] with respect to z and a Laplace transformation [2] with respect to time t. On performing the necessary calculations, we arrive at the following expression for the temperature field in the cylinder:

$$T = \frac{N}{b}\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{\sin(n\alpha)\cos(n\varphi)}{\pi\varepsilon_n n}\,\frac{J_n\left(\frac{\beta_n^m}{a}\rho\right)}{J_{n+1}(\beta_n^m)}[F_n^m + \Phi_n^m]. \tag{3}$$

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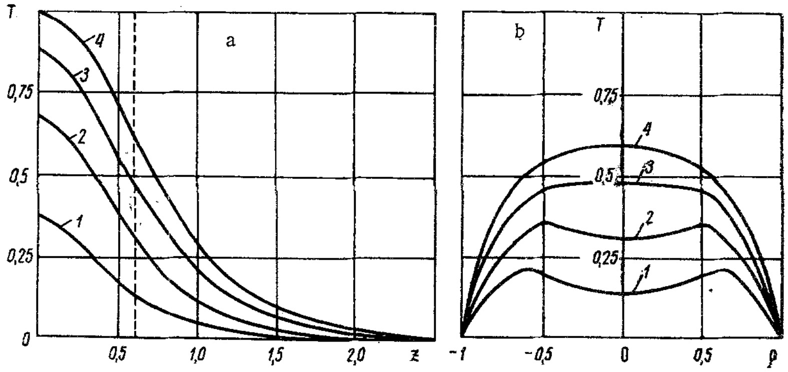

Fig. 1. (a) Axial and (b) radial temperature distributions for various values of time t: 1) t = 1 sec; 2) 1.75; 3) 3.0; 4) t = ∞, a = 1 cm; c = 0.12 cm^2/sec, z = 0.6 (t, sec; ρ, cm; a, cm; c, cm^2/sec; z, cm).

Here

$$F_n^m = \frac{\operatorname{sh}\left(\frac{\beta_n^m}{a}\rho\right)}{\beta_n^m h}\exp\left(-\frac{\beta_n^m}{a}z\right);$$

$$\Phi_n^m = \frac{1}{4a}\sum_{k=0}^{1}\frac{1}{h}\int_0^h\left\{\exp\left[-\frac{\beta_n^m}{a}(z+(-1)^k\theta)\right]\right.$$

$$\times\operatorname{erfc}\left[\sqrt{ct}\,\frac{\beta_n^m}{a}-\frac{z+(-1)^k\theta}{2\sqrt{ct}}\right]+\exp\left[\frac{\beta_n^m}{a}(z+(-1)^k\theta)\right]\operatorname{erfc}\left[\sqrt{ct}\,\frac{\beta_n^m}{a}+\frac{z+(-1)^k\theta}{2\sqrt{ct}}\right]\Bigg\}\,d\theta.$$

Formula (3) simplifies somewhat for an annular source 2b = 2πa for small axial extent, i.e., h ≪ 1. In this case

$$T=\frac{N}{2\pi}\sum_{m=1}^{\infty}\frac{J_0\left(\frac{\beta_0^m}{a}\rho\right)}{J_1(\beta_0^m)}\exp\left(-\frac{\beta_0^m}{a}z\right)\left\{1-\frac{1}{2a}\right.$$

$$\times\left[\operatorname{erfc}\left(\sqrt{ct}\,\frac{\beta_0^m}{a}-\frac{z}{2\sqrt{ct}}\right)+\exp\left(2\frac{\beta_0^m}{a}z\right)\operatorname{erfc}\left(\sqrt{ct}\,\frac{\beta_0^m}{a}+\frac{z}{2\sqrt{ct}}\right)\right]\Bigg\}. \quad (4)$$

The axial and radial temperature distributions calculated through formula (4) are shown in Fig. 1a and b.

As can be seen from the curves, there is a considerable delay in the heating of points ρ = 0 (Fig. 1b) in the initial period following the application of the temperature step. The temperature subsequently builds up to its limiting value.

The thermal fields produced by a pulsed source can be analyzed with the aid of solutions (3) and (4). The technique is described in [3].

Analogous methods were used in [4] to study periodic thermal fields in a cylinder with second-order boundary conditions.

NOTATION

ρ, φ, z	are the cylindrical coordinates;
a	is the radius of the cylinder;
2h, 2b	are the linear dimensions of the source;
T_0	is the temperature of the source;
c	is the thermal conductivity;
t	is the time;
Δ	is the Laplace operator;

J_n is the Bessel function of order n;
β_n^m is the roots of Bessel functions.

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